MATHEMATICS-XII

2016

MATHEMATICS

Section-I (Objective Type)

Time : 1 Hour 10 Minutes] [Marks : 40

Instructions to the Candidates :

1. Fill in your Roll No. in the space provided on the first page of this question paper.
2. This question paper consists of **40** objective type questions. Total marks allotted is **40.**
3. The candidate has to answer all the questions in the **OMR** Answer-Sheet provided along with this question paper.
4. Before answering the candidate has to ensure that the **OMR** Answer-Sheet is available along with the question paper.
5. All entries must be confined to the area provided in the **OMR** Answer-Sheet.
6. Answer all the questions by completely darkening the circles against the question numbers in the **OMR** Answer-Sheet using Black/Blue Ball point pen only.
7. Do not fold or make any stray marks on the OMR Answer Sheet, failing which it would be difficult to evaluate the Answer Sheet.
8. Read all the instructions provided in the OMR Answer-Sheet carefully before answering. After you finish answering, hand over the OMR Answer-Sheet to the Invigilator. You are permitted to carry the question paper only along with you.

For the following Question Nos. 1 to 40 there is only one correct answer against each question. For each question, mark the correct option on the answer sheet.

$40 \times 1 = 40$

1. $f : A \to B$ will be an onto function if

(A) $f(A) \subset B$ (B) $f(A) = B$ (C) $f(A) \supset B$ (D) $f(A) \neq B$

2. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$

3. $\tan^{-1} x + \cot^{-1} x =$

(A) $-\pi$ (B) $-\frac{\pi}{2}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

4. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} =$

(A) $(a+b)(b+c)(c+a)$ (B) $(a+b)(b-c)(c-a)$

(C) $(a-b)(b-c)(c+a)$ (D) $(a-b)(b-c)(c-a)$

5. $A = \begin{bmatrix} 3 & 6 \\ 5 & -4 \end{bmatrix}, B = \begin{bmatrix} 7 & 8 \\ 5 & 6 \end{bmatrix} \Rightarrow 2A + 3B =$

(A) $\begin{bmatrix} 27 & 24 \\ 22 & 10 \end{bmatrix}$ (B) $\begin{bmatrix} 27 & 24 \\ 22 & 10 \end{bmatrix}$ (C) $\begin{bmatrix} 27 & 24 \\ 22 & 10 \end{bmatrix}$ (D) $\begin{bmatrix} 27 & 24 \\ 22 & 10 \end{bmatrix}$

6. $\frac{d}{dx}(\cos^{-1} x) =$

(A) $\frac{1}{2\sqrt{1-x^2}}$ (B) $\sqrt{1-x^2}$ (C) $\frac{-1}{\sqrt{1-x^2}}$ (D) $\frac{1}{\sqrt{1-x^2}}$

7. $\frac{d}{dx}(\tan^{-1} x + \cot^{-1} x)$

(A) $\frac{2}{1+x^2}$ (B) 0 (C) 1 (D) 2

8. If $y = \cos(\log x)$, then $\frac{dy}{dx} =$

(A) $-\sin(\log x)$ (B) $\frac{-\sin(\log x)}{x}$ (C) $\frac{\cos(\log x)}{x}$ (D) $-\sin(\log x)\log x$

9. If $y = x^3$, then $\frac{dy}{dx} =$

(A) $3x^2$ (B) $6x$ (C) 6 (D) 0

10. $\int x^8 dx =$

(A) $8x^7 + k$ (B) $\frac{x^8}{8} + k$ (C) $x^9 + k$ (D) $\frac{x^9}{9} + k$

11. The integration of 0 with respect to x is

(A) 0 (B) k (C) $x + k$ (D) $x^2 + k$

12. $\int \frac{dx}{1 - \sin x} =$

(A) $\tan x - \sec x + k$ (B) $\tan x + \sec x + k$

(C) $\tan^2 x + \sec^2 x + k$ (D) $2(\tan x - \sec x) + k$

13. $\int_a^b x^2 dx =$

(A) $\frac{b^3 - a^3}{3}$ (B) $\frac{a^3 - b^3}{3}$ (C) $\frac{a^2 - b^2}{2}$ (D) $\frac{b^2 - a^2}{2}$

14. The solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(A) $e^x + e^{-y} + k = 0$ (B) $e^{2x} = ke^y$

(C) $e^x = ke^{2y}$ (D) $e^x = ke^y$

15. The integrating factor of the line differential equation $\frac{dy}{dx} + Py = Q$ is

(A) $e^{\int Pdy}$ (B) $e^{\int Qdx}$ (C) $e^{\int Qdy}$ (D) $e^{\int Pdx}$

16. The order of the differential equation $\left(\frac{dy}{dx}\right)^2 + y = x$ is

(A) 0 (B) 1 (C) 2 (D) 3

17. The degree of the equation $\left(\frac{d^2y}{dx^2}\right)^2 - x\left(\frac{dy}{dx}\right)^3 = y^3$ is

(A) 0 (B) 1 (C) 2 (D) 3

18. The position vector of the point (x, y, z) is

(A) $x\vec{i} - y\vec{j} - z\vec{k}$
(B) $x\vec{i} + y\vec{j} - z\vec{k}$
(C) $x\vec{i} - y\vec{j} + z\vec{k}$
(D) $x\vec{i} + y\vec{j} + z\vec{k}$

19. $|-\vec{i} + 2\vec{j} - 3\vec{k}| =$

(A) $\sqrt{15}$ (B) $\sqrt{3}$ (C) 2 (D) $\sqrt{14}$

20. If the position vectors of the point A and B be respectively $(1, 2, 3)$ and $(-3, -4, 0)$ then $\vec{AB} =$

(A) $4\vec{i} + 6\vec{j} + 3\vec{k}$
(B) $-4\vec{a} - 6\vec{j} - 3\vec{k}$
(C) $-3\vec{a} - 8\vec{k}$
(D) $-3\vec{a} + 8\vec{k}$

21. If $\vec{a} = 3\vec{i} + 2\vec{j} + \vec{k}$, $\vec{b} = 4\vec{i} - 5\vec{j} + 3\vec{k}$, then $\vec{a} \cdot \vec{b} =$

(A) 2 (B) 3 (C) 5 (D) 7

22. If $\vec{a}$ and $\vec{b}$ are perpendicular to each other then

(A) $\vec{a} \cdot \vec{b} = 0$ (B) $\vec{a} \times \vec{b} = \vec{0}$ (C) $\vec{a} + \vec{b} = \vec{0}$ (D) $\vec{a} - \vec{b} = 0$

23. $\vec{a} \times \vec{a} =$

(A) 1 (B) 0 (C) α^2 (D) α

24. $\vec{i} \times \vec{j} =$

(A) 0 (B) 1 (C) $\vec{k}$ (D) $-\vec{k}$

25. $\vec{k} \cdot \vec{k} =$

(A) 0 (B) 1 (C) $\vec{i}$ (D) $\vec{j}$

26. The direction cosines of the x-axis are

(A) (0, 0, 0) (B) (1, 0, 0) (C) (0, 1, 0) (D) (0, 0, 1)

27. If l, m, n are the direction cosines of a straight line then

(A) $l^2 + m^2 - n^2 = 1$
(B) $l^2 - m^2 + n^2 = 1$
(C) $l^2 - m^2 - n^2 = 1$
(D) $l^2 + m^2 + n^2 = 1$

28. The distance between the points $(4, 3, 7)$ and $(1, -1, -5)$ is

(A) 7 (B) 12 (C) 13 (D) 25

29. The direction ratios of a straight line are 1, 3, 5. Its direction cosines are

(A) $\frac{1}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{5}{\sqrt{35}}$
(B) $\frac{1}{9}, \frac{1}{3}, \frac{5}{9}$
(C) $\frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}}$
(D) $\frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}}$

30. The direction ratios of two straight lines are l, m, n and l_1, m_1, n_1. The lines will be perpendicular to each other if

(A) $\frac{l}{l_1} = \frac{m}{m_1} = \frac{n}{n_1}$
(B) $\frac{l}{l_1} + \frac{m}{m_1} + \frac{n}{n_1} = 0$
(C) $ll_1 + mm_1 + nn_1 = 0$
(D) $ll_1 + mm_1 + nn_1 = 1$

31. A line passing through (2, – 1, 3) and its direction ratios are 3, – 1, 2. The equation of the line is

(A) $\frac{x+2}{3}=\frac{y-1}{-1}=\frac{z+3}{2}$ (B) $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$

(C) $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z-2}{3}$ (D) $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z-2}{3}$

32. The lines $\frac{x-1}{l}=\frac{y+2}{m}=\frac{z-4}{n}$ and $\frac{x+3}{2}=\frac{y-4}{3}=\frac{z}{6}$ are parallel to each other if

(A) $2l=3m=n$ (B) $3l=2m=n$

(C) $2l+3m+6n=0$ (D) $lmn=36$

33. The length of the perpendicular from the point (0, – 1, 3) to the plane $2x+y-2z+1=0$ is

(A) 0 (B) $2\sqrt{3}$ (C) 2/3 (D) 2

34. If $P(A)=\frac{3}{8}$, $P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{4}$, then $P\left(\frac{A}{B}\right)=$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{8}$

35. If A and B are two independent events, then

(A) $P(AB')=P(A)P(B)$ (B) $P(AB')=P(A)P(B')$

(C) $P(AB')=P(A')+P(B)$ (D) $P(AB')=P(A)+P(B')$

36. The matrix $\begin{bmatrix}3 & 5\\ 2 & k\end{bmatrix}$ has no inverse if the value of k s

(A) 0 (B) 5 (C) $\frac{10}{3}$ (D) $\frac{4}{9}$

37. $\begin{vmatrix}2 & 3 & 5\\ 0 & 4 & 7\\ 0 & 0 & 5\end{vmatrix}=$

(A) 40 (B) 0 (C) 3 (D) 25

38. $\tan^{-1}\frac{2x}{1-x^2}=$

(A) $2\sin^{-1}x$ (B) $\sin^{-1}2x$ (C) $\tan^{-1}2x$ (D) $2\tan^{-1}x$

39. $\int\frac{-1}{1+x^2}dx=$

(A) $\tan^{-1}x+k$ (B) $\sec^{-1}x+k$

(C) $\text{cosec}^{-1}x+k$ (D) $\cot^{-1}x+k$

40. $\int\frac{dx}{x^2+a^2}=$

(A) $\frac{1}{a}\tan^{-1}\frac{x}{a}+k$ (B) $\frac{1}{a}\tan^{-1}(x+a)+k$

(C) $\sin^{-1}\frac{x}{a}+k$ (D) $\cos^{-1}\frac{x}{a}+k$

1. (B) 2. (C)
8. (B) 9. (B)
15. (D) 16. (B)
22. (A) 23. (B)
29. (A) 30. (C)
36. (C) 37. (A)

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Time : 2 Hour 05 Minutes]

Instructions to the Candi

1. Candidates are requ
2. Figures in the right-
3. Section II of this qu total marks 60.
4. The candidate has to a long answer type quest separately. Q.Nos. 1 to
5. Write the question n

Question Nos. 1 to 8 are o

1. Prove that 4 (cot^{-1}

Soln. L.H.S. = 4 (cot

= 4 (tan

= 4 tan^{-}

= 4 tan^{-}

2. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ then fi

Soln. $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ ∴

Now, $A^2+3A+2I=$

ANSWERS

1. (B)	2. (C)	3. (B)	4. (D)	5. (B)	6. (C)	7. (B)
8. (B)	9. (B)	10. (D)	11. (B)	12. (B)	13. (B)	14. (A)
15. (D)	16. (B)	17. (C)	18. (D)	19. (D)	20. (B)	21. (C)
22. (A)	23. (B)	24. (C)	25. (B)	26. (B)	27. (D)	28. (C)
29. (A)	30. (C)	31. (B)	32. (B)	33. (D)	34. (B)	35. (B)
36. (C)	37. (A)	38. (D)	39. (D)	40. (A)		

Section-II (Non-Objective Type)

Time : 2 Hour 05 Minutes] [Marks : 60

Instructions to the Candidates :

1. Candidates are required to give their answers in their own words as far as practicable.
2. Figures in the right-hand margin indicate full marks.
3. Section II of this question paper consists of 12 non-objective type questions having total marks 60.
4. The candidate has to answer all the short answer questions from Q. No. 1 to Q. No. 8 and all 4 long answer type questions from Q. No. 9 to Q. No. 12 in his/her answer-book which is provided separately. Q.Nos. 1 to 8 carry 4 marks each and Q. Nos. 9 to 12 carry 7 marks each.
5. Write the question number with every answer.

Question Nos. 1 to 8 are of short answer type. Each question carries 4 marks.

$8 \times 4 = 32$

Short Answer Type Questions

1. Prove that $4\,(\cot^{-1} 3 \pm \operatorname{cosec}^{-1}\sqrt{5}) = \pi$.

Soln. L.H.S. $= 4\,(\cot^{-1} 3 + \operatorname{cosec}^{-1}\sqrt{5})$

$$= 4\left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}\right) \quad \left[\because \tan^{-1}x = \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= 4\tan^{-1}\left(\frac{5}{6} \times \frac{6}{5}\right) = 4\tan^{-1}(1)$$

$$= 4\tan^{-1}\left(\tan\frac{\pi}{4}\right) = 4 \cdot \frac{\pi}{4} = \pi.$$

2. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find the value of $A^2 + 3A + 2I$.

Soln. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\therefore A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$

Now, $A^2 + 3A + 2I = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 16 \\ 24 & 36 \end{bmatrix}.$$

3. Evaluate : $\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$.

Soln. Taking x, y, z as common from R_1, R_2 and R_3 respectively, we get

$$xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \qquad [R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3]$$

$$= xyz\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix} = xyz\begin{vmatrix} 0 & (x-y) & (x-y)(x+y) \\ 0 & (y-z) & (y-z)(y+z) \\ 1 & z & z^2 \end{vmatrix}.$$

Taking common as $(x-y)$ from R_1 and $(y-z)$ from R_2, we get

$$=(xyz)(x-y)(y-z)\begin{vmatrix} 0 & 1 & x+y \\ 0 & 1 & y+z \\ 1 & z & z^2 \end{vmatrix} \qquad \text{[Expanding along } C_1\text{]}$$

$$=(xyz)(x-y)(y-z)\times 1\begin{vmatrix} 1 & x+y \\ 1 & y+z \end{vmatrix}$$

$$=(x-y)(y-z)(xyz)(y+z-x-y)=(x-y)(y-z)(z-x)(xyz) \text{ Ans.}$$

4. Solve for x : $\tan^{-1}2x + \tan^{-1}3x = \dfrac{\pi}{4}$.

Soln. Given $\tan^{-1}\dfrac{2x+3x}{1-2x\cdot 3x} = \dfrac{\pi}{4}$; because $6x^2 < 1$

$\Rightarrow\ x^2 < \dfrac{1}{6} \Rightarrow -\dfrac{1}{\sqrt{6}} < x < \dfrac{1}{\sqrt{6}}$ ...(A)

$\Rightarrow\ \tan^{-1}\dfrac{5x}{1-6x^2} = \dfrac{\pi}{4} \Rightarrow \dfrac{5x}{1-6x^2} = \tan\dfrac{\pi}{4} = 1$

$\Rightarrow\ 5x = 1 - 6x^2 \Rightarrow 6x^2 + 5x - 1 = 0 \Rightarrow (x+1)(6x-1) = 0$

$\Rightarrow\ x + 1 = 0$ ... (1) or, $6x - 1 = 0$ ... (2)

From (1), $x = -1$ not satisfied condition (A)

From (2), $6x = 1 \Rightarrow x = \dfrac{1}{6}$ satisfied condition (A)

Hence solution of given eqaution is $x = \dfrac{1}{6}$.

5. If $y = \sin[\cos\{\tan(\sin^{-1}x)\}]$, then find $\dfrac{dy}{dx}$.

Soln. $\dfrac{dy}{dx} = \cos[\cos\{\tan(\sin^{-1}x)\}]\cdot(-)\sin\{\tan(\sin^{-1}x)\}\cdot\sec^2(\sin^{-1}x)\cdot\dfrac{1}{\sqrt{1-x^2}}$

$= \dfrac{(-)\cos[\cos\{\tan(\sin^{-1}x)\}]\cdot\sin\{\tan(\sin^{-1}x)\cdot\sec^2(\sin^{-1}x)}{\sqrt{1-x^2}}$.

6. Integrate : $\int e^x \cos x\, dx$.

Soln. Let $I = \int e^x \cos x\, dx$.

$= \cos x \cdot e^x + \int \sin x \cdot e^x \, dx = e^x \cos x + \sin x \cdot e^x - \int \cos x \, e^x \, dx$

$I = e^x(\sin x + \cos x) - I$

$\therefore \quad I = \frac{e^x}{2}(\sin x + \cos x).$

7. If $\vec{a} = 2\hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{b} = -7\hat{i} + 6\hat{j} + 8\hat{k}$ then find $\vec{a} \times \vec{b}$.

Soln. $\vec{a} = 2\hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{b} = -7\hat{i} + 6\hat{j} + 8\hat{k}$

$$\therefore \quad \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -5 \\ -7 & 6 & 8 \end{vmatrix} = \hat{i}(-24 + 30) - \hat{j}(16 - 35) + \hat{k}(12 - 21)$$
$$= 6\hat{i} + 19\hat{j} - 9\hat{k}.$$

8. What is the chance of getting 7 or 11 with two dice?

Soln. $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$

... ...

... ...

$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$

$n(S) = 36$

$E_1 = \text{getting } 7 = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$n(E_1) = 6$

$E_2 = \text{getting } 11 = \{(5, 6), (6, 5)\}$

$\therefore E_1$ and E_2 mutually exclusive.

$\therefore \quad E = E_1 + E_2 = 6 + 2 = 8 \;\therefore\; P(E) = \frac{n(E)}{n(S)} = \frac{8}{36} = \frac{2}{9}$ **Ans.**

Question Nos. 9 to 12 are of long answer type. Each question carries 7 marks.

$4 \times 7 = 28$

Long Answer Type Questions

9. Solve : $\frac{dy}{dx} - \frac{2y}{x} = y^4$

Soln. Dividing both sides by y^4

$$\frac{1}{y^4}\frac{dy}{dx} - \frac{2}{xy^3} = 1 \quad \text{... (1)}$$

Put $\frac{1}{y^3} = z \Rightarrow y^{-3} = z \Rightarrow -3y^{-4}\frac{dy}{dx} = \frac{dz}{dx} \Rightarrow \frac{1}{y^4}\frac{dy}{dx} = -\frac{1}{3}\frac{dz}{dx}.$

From (1), $-\frac{1}{3}\frac{dz}{dx} - \frac{2}{x} \cdot z = 1$

$\Rightarrow \quad \frac{dz}{dx} + \frac{6z}{x} = -3 \Rightarrow \frac{dz}{dx} + Pz = Q$, where $P = \frac{6}{x}, Q = -3$

I.F. $= e^{\int P dx} = e^{\int \frac{6}{x} dx} = e^{6\int \frac{1}{x} dx} = e^{6 \log x} = x^6.$

$\therefore$ Solution of L.D.E. is $z \cdot \text{I.F.} = \int (Q \cdot \text{I.F.})\, dx$

$\Rightarrow \quad z \cdot x^6 = \int 3 \cdot x^6 dx \Rightarrow \frac{1}{y^3} \cdot x^6 = -3\frac{x^7}{7} + C \;\therefore\; \frac{x^6}{y^3} + \frac{3x^7}{7} = C$

Or,

Solve : $y^2dx + (x^2 + xy)dy = 0$.

Soln. $y^2dx + (x^2 + xy)dy = 0 \Rightarrow \frac{dy}{dx} = \frac{-y^2}{xy + x^2}$ ...(1)

Put $y = vx$ $\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

From (1), $v + x\frac{dv}{dx} = -\frac{v^2x^2}{x^2(1+v)} \Rightarrow x\frac{dv}{dx} = \frac{-2v^2 - v}{v+1}$

$\Rightarrow \int \frac{v+1}{2v^2+1}dv = \int -\frac{dx}{x} \Rightarrow \int \frac{v+1}{(v(2v+1)}dv = -\int \frac{dx}{x}$ ...(2)

Now, $\frac{v+1}{v(2v+1)} = \frac{A}{v} + \frac{B}{2v+1}$

$\Rightarrow \frac{v+1}{v(2v+1)} = \frac{A(2v+1) + Bv}{v(2v+1)}$

$\Rightarrow v + 1 = A(2v + 1) + Bv.$

We get $A = 1$ and $B = -1$

From (2), $\int \left(\frac{A}{v} + \frac{B}{2v+1}\right)dv = -\int \frac{dx}{x}$

$\Rightarrow \log v - \frac{\log(2v+1)}{2} = -\log x + \log c$

$\Rightarrow 2\log v - \log(2v+1) = -2\log x + 2\log c$

$\Rightarrow \log \frac{v^2}{2v+1} = \log c^2 - \log x^2$

$\Rightarrow \log\left(\frac{y^2}{x^2 + 2xy}\right) = \log \frac{c^2}{x^2}$

$\Rightarrow \frac{y^2}{x(x+2y)} = \frac{c^2}{x^2} \Rightarrow xy^2 = c^2(x + 2y)$

where c is constant of integration.

10. Prove that $\int_0^{\pi/2} \log \sin x\, dx = \int_0^{\pi/2} \log \cos x\, dx = -\frac{\pi}{2}\log 2$.

Soln. Let $I = \int_0^{\pi/2} \log \sin x dx$ ...(1)

Then $I = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right)dx;$ since $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$= \int_0^{\pi/2} \log \cos x dx$ ...(2)

(1) + (2) $\Rightarrow 2I = \int_0^{\pi/2} (\log \sin x + \log \cos x)dx$

$= \int_0^{\pi/2} \log(\sin x \cos x)dx$

$= \int_0^{\pi/2} \log \frac{\sin 2x}{2} dx$

$$= \int_0^{\pi/2} \{\log \sin 2x - \log 2\}\, dx$$

$$= \int_0^{\pi/2} \log \sin 2x dx - (\log 2) \int_0^{\pi/2} dx$$

$$= I_1 - \frac{\pi}{2} \log 2$$

Now we find the value of $I_1 = \int_0^{\pi/2} \log \sin 2x dx$ ... (3)

Let $2x = t$ in which $2dx = dt$.

Also, $(x = 0 \Rightarrow t = 0)$ and $(x = \pi/2 \Rightarrow t = \pi)$

$$\Rightarrow \quad I_1 = \int_0^{\pi} \log \sin t \cdot \frac{dt}{2} = \frac{1}{2} \int_0^{\pi} \log \sin t dt.$$

Here $\log \sin \left(2 \cdot \frac{\pi}{2} - t\right) = \log \sin t \; \therefore \; I_1 = \frac{1}{2} \cdot 2 \int_0^{\pi/2} \log \sin t dt;$

But if $f(2a - x) = f(x)$, then $\int_0^{2a} f(x) dx = 2 \int_0^{a} f(x) dx.$

Hence $I_1 = \int_0^{\pi/2} \log \sin t dt = \int_0^{\pi/2} \log \sin x dx = I.$

$\therefore$ From (1), $2I = I - \frac{\pi}{2} \log 2 \Rightarrow I = -\frac{\pi}{2} \log 2.$

11. Find the co-ordinates of the point where the line joining the points $P(1, -2, 3)$ and $Q(4, 7, 8)$ cuts the xy-plane.

Soln. Equation of xy plane is $z = 0$

The equation of line is $\frac{x-1}{3} = \frac{y+2}{9} = \frac{z-3}{5}$

any point on this ine is $(1 + 3\lambda, -2 + 9\lambda, 3 + 5\lambda)$ this line lies on the xy-plane

$\Rightarrow \; 3 + 5\lambda = 0 \Rightarrow 5\lambda = -3 \therefore \lambda = -3/5$

Required point is $\left(1 + 3\left(-\frac{3}{6}\right), -2 + 9\left(-\frac{3}{5}\right), 0\right)$

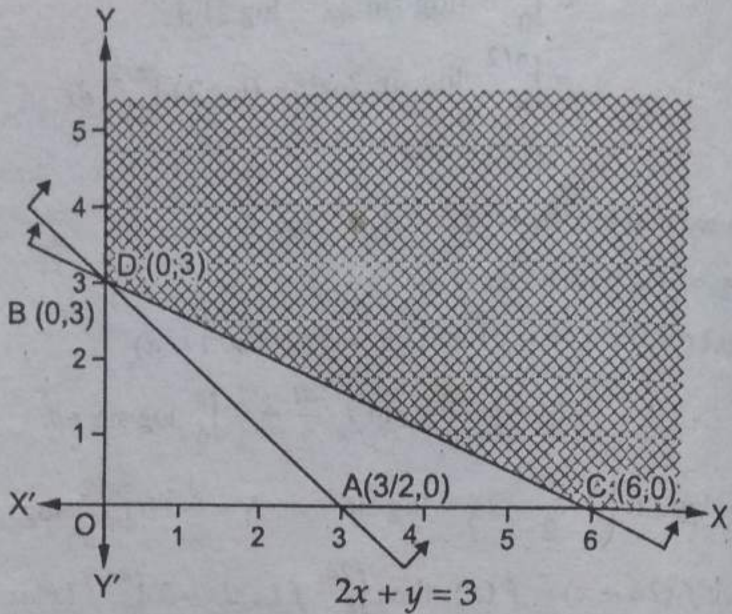
$$= \left(1 - \frac{9}{5}, \; -2 - \frac{27}{5}, 0\right) = \left(-\frac{4}{5}, -\frac{37}{5}, 0\right).$$

12. Minimize : $Z = x + 2y$
subject to $2x + y \geq 3$
$x + 2y \geq 6$
$x, y \geq 0.$

Soln. First all, let us graph the feasible region of the given system of inequations. The shaded region in the above figure is the feasible region. We observe that the feasible region is unbounded. We now evaluate Z at the corner points.

Corner point	$Z = x + 2y$
C(6, 0)	6
B, D(0, 3)	6

From the table, we find that 6 is the smallest value of Z at the corner point $(6, 0)$ as well as at the corner point $(0, 3)$.



Since the feasible region is unbounded, 6 may or may not be the minimum value of Z. To decide this issue, we graph the inequation $x + 2y < 6$ (step 3(b) of corner point method) and check whether the resulting open half plane has points in common with feasible region or not.

If it has common points then 6 will not be the minimum value of Z; otherwise 6 will be the minimum value of Z. It can be shown in the figure that it has no common points (because the region below the line $x + 2y = 6$ represents the inequation $x + 2y < 6$).

Hence Z has a minimum value 6 at C as well as at $D(B)$ i.e., Z has a minimum value 6 at all points on the line segment CD.

□

Time : 1 Hour 10 Minutes]
Instructions to the Candid...
For the following Questi...
each question. For each q...

1. If $f(x_1) = f(x_2) \Rightarrow$
(A) one-one (B)

2. The principal value of
(A) $\frac{2\pi}{3}$ (B)

3. $\tan^{-1} x =$
(A) $\cot^{-1} x$ (B)

4. $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} =$
(A) $(x-y)(y+z)(z+x)$
(C) $(x-y)(y-z)(z+x)$

5. If $\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & \\ 3 & \end{bmatrix}$
(A) 3 (B) 4

6. $\frac{d}{dx}(\sin^{-1} x) =$
(A) $\frac{1}{\sqrt{1-x^2}}$ (B) –

7. $\frac{d}{dx}(\sin^{-1} x + \cos^{-1} x) =$
(A) 0 (B) 1

8. If $y = \sin(x^3)$, then $\frac{dy}{dx} =$
(A) $x^3 \cos(x^3)$ (B) $3x^2$

9. If $y = \tan^2 x$, then $\frac{dy}{dx} =$
(A) $\sec^2 x$ (B) $\sec^4$

10. $\int 1.\, dx =$
(A) $x + k$