2015

MATHEMATICS

Section-I (Objective Type)

Time : 1 Hour 10 Minutes] [Marks : 40

Instructions to the Candidates : See Question Paper 2016.

For the following Question Nos. 1 to 40 there is only one correct answer against each question. For each question, mark the correct option on the answer sheet.

$40 \times 1 = 40$

1. If $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \ \forall x_1\ x_2 \in A$, then the function $f : A \to B$ is

(A) one-one (B) constant (C) onto (D) many one

2. The principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

3. $\tan^{-1} x =$

(A) $\cot^{-1} x$ (B) $\frac{1}{\cot^{-1} x}$ (C) $\cot^{-1}\frac{1}{x}$ (D) $-\cot^{-1} x$

4. $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} =$

(A) $(x-y)(y+z)(z+x)$ (B) $(x+y)(y-z)(z-x)$

(C) $(x-y)(y-z)(z+x)$ (D) $(x-y)(y-z)(z-x)$

5. If $\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$, then $x =$

(A) 3 (B) 4 (C) 5 (D) 8

6. $\frac{d}{dx}(\sin^{-1} x) =$

(A) $\frac{1}{\sqrt{1-x^2}}$ (B) $-\frac{1}{\sqrt{1-x^2}}$ (C) $2(1-x^2)$ (D) $(1-x^2)$

7. $\frac{d}{dx}(\sin^{-1} x + \cos^{-1} x) =$

(A) 0 (B) 1 (C) $\frac{\pi}{2}$ (D) $\frac{1}{\sqrt{1-x^2}}$

8. If $y = \sin(x^3)$, then $\frac{dy}{dx} =$

(A) $x^3 \cos(x^3)$ (B) $3x^2 \sin(x^3)$ (C) $3x^2 \cos(x^3)$ (D) $\cos(x^3)$

9. If $y = \tan^2 x$, then $\frac{dy}{dx} =$

(A) $\sec^2 x$ (B) $\sec^4 x$ (C) $2 \tan x \sec x$ (D) $2 \tan x \sec^2 x$

10. $\int 1 \cdot dx =$

(A) $x + k$ (B) $1 + k$ (C) $\frac{x^2}{2} + k$ (D) $\log x + k$

11. $\int \frac{dx}{\sqrt{x}} =$

(A) $\sqrt{x} + k$ (B) $2\sqrt{x} + k$ (C) $x + k$ (D) $\frac{2}{3}x^{3/2} + k$

12. $\int \frac{dx}{1 + \cos x} =$

(A) $\tan\frac{x}{2} + k$ (B) $\frac{1}{2}\tan\frac{x}{2} + k$ (C) $2\tan\frac{x}{2} + k$ (D) $\tan^2\frac{x}{2} + k$

13. $\int_a^b x^5 dx =$

(A) $b^5 - a^5$ (B) $\frac{b^6 - a^6}{6}$ (C) $\frac{a^6 - b^6}{6}$ (D) $a^5 - b^5$

14. The solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$ is

(A) $x - y = k$ (B) $x^2 - y^2 = k$ (C) $x^3 - y^3 = k$ (D) $xy = k$

15. The integrating factor of the linear differential equation $\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$ is

(A) $\tan x$ (B) $e^{\tan x}$ (C) $\log \tan x$ (D) $\tan^2 x$

16. The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = \frac{d^2y}{dx^2}$ is

(A) 1 (B) 2 (C) 3 (D) 4

17. The order of the differential equation $\frac{d^2y}{dx^2} + x^3\left(\frac{dy}{dx}\right)^3 = x^4$ is

(A) 1 (B) 2 (C) 3 (D) 4

18. The position vector of the point (1, 0, 2) is

(A) $\vec{i} + \vec{j} + 2\vec{k}$ (B) $\vec{i} + 2\vec{j}$ (C) $\vec{i} + 3\vec{k}$ (D) $\vec{i} + 2\vec{k}$

19. The modulus of $7\vec{i} - 2\vec{j} + \vec{k}$ is

(A) $\sqrt{10}$ (B) $\sqrt{55}$ (C) $3\sqrt{6}$ (D) 6

20. If O be the origin and $\vec{OP} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{OQ} = 5\hat{i} + 4\hat{j} - 3\hat{k}$, then $\vec{PQ}$ equal to

(A) $7\hat{i} + 7\hat{j} + 7\hat{k}$ (B) $-3\hat{i} - \hat{j} - \hat{k}$

(C) $-7\hat{i} + 7\hat{j} + 7\hat{k}$ (D) $3\hat{i} - \hat{j} - \hat{k}$

21. The scalar product of $5\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} - 4\hat{j} + 7\hat{k}$ is

(A) 10 (B) – 10 (C) 15 (D) – 15

22. If $\vec{a} \cdot \vec{b} = 0$ then

(A) $\vec{a} \perp \vec{b}$ (B) $\vec{a} \parallel \vec{b}$ (C) $\vec{a} + \vec{b} = 0$ (D) $\vec{a} - \vec{b} = \vec{0}$

23. $\vec{i} \cdot \vec{j} =$

(A) 0 (B) 1 (C) $\vec{k}$ (D) $-\vec{k}$

24. $\vec{k} \times \vec{j} =$

(A) 0 (B) 1 (C) $\vec{i}$ (D) $-\vec{i}$

25. $\vec{a} \cdot \vec{a} =$

(A) 0 (B) 1 (C) $|\vec{a}|^2$ (D) $|\vec{a}|$

26. **The direction cosines of the y-axis are**

(A) (0, 0, 0) (B) (1, 0, 0) (C) (0, 1, 0) (D) (0, 0, 1)

27. **The direction ratios of the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are**

(A) $x_1 + x_2, y_1 + y_2, z_1 + z_2$

(B) $\sqrt{(x_1 - x_2^2) + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

(C) $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}$

(D) $x_2 - x_1, y_2 - y_1, z_2 - z_1$

28. **The coordinates of the midpoint of the line segment joining the points (2,3, 4) and (8, –3, 8) are**

(A) (10, 0, 12) (B) (5, 6, 0) (C) (6, 5, 0) (D) (5, 0, 6)

29. **If the direction cosines of two straight lines are l_1, m_1, n_1 and l_2, m_2, n_2 then the cosine of the angle θ between them or cos θ is**

(A) $(l_1 + m_1 + n_1)(l_2 + m_2 + n_2)$

(B) $\frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$

(C) $l_1 l_2 + m_1 m_2 + n_1 n_2$

(D) $\frac{l_1 + m_1 + n_1}{l_2 + m_2 + n_2}$

30. **The direction ratios of the normal to the plane $7x + 4y - 2z + 5 = 0$ are**

(A) 7, 4, 5 (B) 7, 4, –2 (C) 7, 4, 2 (D) 0, 0, 0

31. **If the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ is parallel to the plane $ax + by + cz + d = 0$, then**

(A) $\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$

(B) $al + bm + cn = 0$

(C) $al_2 + bm^2 + cn^2 = 0$

(D) $a^2l^2 + b^2m^2 + c^2n^2 = 0$

32. **If the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular to each other, then**

(A) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(B) $\frac{a_1}{a_2} + \frac{b_1}{b_2} + \frac{c_1}{c_2} = 0$

(C) $a_1a_2 + b_1b_2 + c_1c_2 = 0$

(D) $a_1^2a_2^2 + b_1^2b_2^2 + c_1^2c_2^2 = 0$

33. **The distance of the plane $2x - 3y + 6z + 7 = 0$ from the point (2, –3, –1) is**

(A) 4 (B) 3 (C) 2 (D) $\frac{1}{5}$

34. **If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{4}$, then $P(A \cup B) =$**

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$

35. **If A and B are two independent events, then**

(A) $P(A \cup B) = 1 - P(A')P(B')$

(B) $P(A \cap B) = 1 - P(A')P(B')$

(C) $P(A \cup B) = 1 + P(A')P(B')$

(D) $P(A \cup B) = \frac{P(A')}{P(B')}$

36. $\begin{vmatrix} 3 & 4 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 7 \end{vmatrix} =$

(A) 40 (B) 50 (C) 42 (D) 15

37. The inverse of $A = \begin{bmatrix} 2 & 3 \\ 5 & k \end{bmatrix}$ will not be obtained if k has the value

(A) 2 (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{15}{2}$

38. $\cos^{-1}\frac{1-x^2}{1+x^2} =$

(A) $2\cos^{-1}x$ (B) $2\sin^{-1}x$ (C) $2\tan^{-1}x$ (D) $\cos^{-1}2x$

39. For any unit matrix I

(A) $I_2 = I$ (B) $|I| = 0$ (C) $|I| = 2$ (D) $|I| = 5$

40. If $x > a$, $\int \frac{dx}{x^2 - a^2} =$

(A) $\frac{1}{2a}\log\frac{x-a}{x+a} + k$ (B) $\frac{1}{2a}\log\frac{x+a}{x-a} + k$

(C) $\frac{1}{a}\log(x^2 - a^2) + k$ (D) $\log(x + \sqrt{x^2 - a^2}) + k$

ANSWERS

1. (A)	2. (D)	3. (C)	4. (D)	5. none	6. (A)	7. (A)
8. (C)	9. (D)	10. (A)	11. (B)	12. (A)	13. (B)	14. (B)
15. (B)	16. (B)	17. (B)	18. (D)	19. (C)	20. (D)	21. (B)
22. (A)	23. (A)	24. (D)	25. (C)	26. (C)	27. (B)	28. (D)
29. (C)	30. (B)	31. (B)	32. (C)	33. (C)	34. (D)	35. (A)
36. (C)	37. (D)	38. (C)	39. (A)	40. (A)		

Section-II (Non-Objective Type)

Time : 2 Hour 05 Minutes] [Marks : 60

Instructions to the Candidates : See Question Paper 2016.

Question Nos. 1 to 8 are of short answer type. Each question carries 4 marks.

$8 \times 4 = 32$

Short Answer Type Questions

1. Prove that $4(\cot^{-1} 3 + \operatorname{cosec}^{-1}\sqrt{5}) = \pi$.

Soln. L.H.S. $= 4(\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5})$

$= 4\left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}\right)$ $\left[\because \tan^{-1}x = \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= 4\tan^{-1}\left(\frac{5}{6}\times\frac{6}{5}\right) = 4\tan^{-1}(1)$

$= 2\cdot 2\tan^{-1}(1) = 4\tan^{-1}(1) = 4\tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$= 4 \times \frac{\pi}{4} = \pi$..

2. Evaluate : $A = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$.

Soln. Applying $C_2 \to C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} 1 & bc+ca+ab & a(b+c) \\ 1 & bc+ca+ab & b(c+a) \\ 1 & bc+ca+ab & c(a+b) \end{vmatrix} = (bc+ca+ab)\begin{vmatrix} 1 & 1 & a(b+c) \\ 1 & 1 & b(c+a) \\ 1 & 1 & c(a+b) \end{vmatrix}$$

$= (bc + ca + ab) \times 0$; Since C_1 and C_2 are identical

$= 0$

3. Find the values of X and Y : $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

Soln. We have, $(X + Y) + (X - Y)$

$$= \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \Rightarrow X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 10/2 & 0 \\ 2/2 & 8/2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}.$$

Again, $(X + Y) - (X - Y) = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$$\Rightarrow 2Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 7-3 & 0-0 \\ 2+0 & 5-3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}.$$

4. If $y = \sin[\cos\{\tan(\cot x)\}]$, then find $\frac{dy}{dx}$.

Soln. $\because y = \sin[\cos\{\tan(\cot x)\}]$

$$\therefore \frac{dy}{dx} = \frac{d[\sin\{\cos\{\tan(\cot x)\}]}{dx}$$

$$= \frac{d[\sin\{\cos\{\tan(\cot x)\}\}]}{d[\cos\{\tan(\cot x)\}]} \times \frac{d[\cos\{\tan(\cot x)\}]}{d[\tan(\cot x)]} \times \frac{d[\tan(\cot x)]}{d[\cot x]} \times \frac{d[\cot x]}{dx}$$

$= \cos\{\cos\{\tan(\cot x)\}\} \times -\sin\{\tan(\cot x)\} \times \sec^2(\cot x) \times -\operatorname{cosec}^2 x$

$= \cos\{\cos\{\tan(\cot x)\}\} \cdot \sin\{\tan(\cot x)\} \cdot \sec^2(\cot x) \cdot \operatorname{cosec}^2 x.$

5. If $y = \tan^{-1} x$, then find $\frac{dy}{dx}$ by first principle.

Soln. $\because y = \tan^{-1} x \quad \therefore x = \tan y$ ... (i)

Now $(x + \delta x) = \tan(y + \delta y)$ ... (ii)

Substracting equation (i) from equation (ii), then

$$(x + \delta x) - (x) = \tan(y + \delta y) - \tan y$$

$$\Rightarrow \lim_{\delta y \to 0} \frac{\delta x}{\delta y} = \lim_{\delta y \to 0} \frac{\tan(y + \delta y) - \tan y}{\delta y}$$

$$\therefore \frac{dx}{dy} = \lim_{\delta y \to 0} \frac{\dfrac{\sin (y+\delta y)}{\cos (y+\delta y)} - \dfrac{\sin y}{\cos y}}{\delta y}$$

$$= \lim_{\delta y \to 0} \frac{\sin (y+\delta y)\cos y - \cos (y+\delta y)\sin y}{\delta y \cdot \cos (y+\delta y)\cos y}$$

$$= \lim_{\delta y \to 0} \frac{\sin [y+\delta y - y]}{\delta y \cdot \cos (y+\delta y)\cdot \cos y} = \lim_{\delta y \to 0} \frac{\sin \delta y}{\delta y} \times \frac{1}{\cos (y+\delta y)} \times \frac{1}{\cos y}$$

$$= 1 \times \frac{1}{\cos (y+0)} \times \frac{1}{\cos y} = \frac{1}{\cos^2 y} = \sec^2 y = 1 + \tan^2 y = 1 + x^2$$

$$\text{or, } \frac{dy}{dx} = \frac{1}{1+x^2} \quad \therefore \quad \frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}.$$

6. Integrate : $\int e^x \cos x \, dx$.

Soln. Here $I = \int e^x \cos x \, dx$

$$= \cos x \int e^x \, dx - \int \left\{ \frac{d(\cos x)}{dx} \int e^x \, dx \right\} dx$$

$$= \cos x \cdot e^x + \int \sin x \cdot e^x \, dx$$

$$= \cos x \cdot e^x + \left[\sin x \int e^x \cdot dx - \int \left\{ \frac{d(\sin x)}{dx} \int e^x dx \right\} dx \right]$$

$$= \cos x \cdot e^x + [\sin x \cdot e^x - \int \cos x \cdot e^x \, dx$$

$$= \cos x \cdot e^x + \sin x \cdot e^x - I + C$$

$$\Rightarrow \quad 2I = e^x (\cos x + \sin x) + C \quad \therefore \ I = \frac{1}{2} e^x (\cos x + \sin x) + C.$$

7. If $\vec{a} = 2\hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{b} = -7\hat{i} - 6\hat{j} + 8\hat{k}$ then find $\vec{a} \times \vec{b}$

Soln. $\because \vec{a} = 2\hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{b} = -7\hat{i} - 6\hat{j} + 8\hat{k}$

$$\therefore \quad \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -5 \\ -7 & 6 & 8 \end{vmatrix} = \hat{i} \begin{vmatrix} -3 & -5 \\ 6 & 8 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & -5 \\ -7 & 8 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & -3 \\ -7 & 6 \end{vmatrix}$$

$$= \hat{i}(-24 + 30) - \hat{j}(16 - 35) + \hat{k}(12 - 21) = 6\hat{i} + 19\hat{j} - 9\hat{k}.$$

8. A speaks the truth in 75% cases and B in 80% of the cases. In what percentage the cases are they likely to contradict each other in stating the same fact?

Soln. A and B will contradict each other only in that case when one speaks the truth the other does not speak the truth.

Let E_1 = the event when A speaks the truth

E_2 = the event when B speaks the truth.

Then E_1' = the event when A tells a lie

and E_2' = the event when B tells a lie.

Given $P(E_1) = \frac{75}{100} = \frac{3}{4}, P(E_2) = \frac{80}{100} = \frac{4}{5}$

$\therefore \quad P(E_1') = 1 - \frac{3}{4} = \frac{1}{4}, P(E_2') = 1 - \frac{4}{5} = \frac{1}{5}.$

(i) The probability that A speaks the truth and B speaks a lie is

$$P(E_1 \cap E_2') = P(E_1) \cdot P(E_2') = \frac{3}{4} \times \frac{1}{5} = \frac{3}{20}.$$

(ii) The probability that B speaks the truth and A speaks a lie is

$$P(E_2 \cap E_1') = P(E_2) \cdot P(E_1') = \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}.$$

$\because$ (i) and (ii) are mutually exclusive, therefore the required probability $= \frac{3}{20} + \frac{1}{5} = \frac{7}{20}$.

$\therefore$ The percentage when they contradict each other = 35%.

Question Nos. 9 to 12 are of long answer type. Each question carries 7 marks.

$4 \times 7 = 28$

Long Answer Type Questions

9. Evaluate $\int_0^{\pi/4} \sin^2 x \, dx$.

Soln. $I = \int_0^{\pi/4} \sin^2 x \, dx = \frac{1}{2} \int_0^{\pi/4} 2\sin^2 x \, dx$

$$= \frac{1}{2} \int_0^{\pi/4} (1 - \cos 2x) \, dx = \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\pi/4}$$

$$= \frac{1}{2} \left[\left\{ \frac{\pi}{4} - \frac{1}{2} \sin \frac{\pi}{2} \right\} - \left\{ 0 - \frac{1}{2} \sin 0 \right\} \right]$$

$$= \frac{1}{2} \left[\frac{\pi}{4} - \frac{1}{2} \times 1 + \frac{1}{2} \times 0 \right] = \frac{1}{2} \left[\frac{\pi}{4} - \frac{1}{2} \right] = \frac{\pi}{8} - \frac{1}{4}.$$

10. Solve the differential equation : $(x - y)dy - (x + y)dx = 0$.

Soln. The given differential equation can be written as

$$(x - y)dy = (x + y)dx \text{ i.e., } \frac{dy}{dx} = \frac{x + y}{x - y} \qquad \dots (1)$$

This is of the form $\frac{dy}{dx} = F(x, y)$, where $F(x, y) = \frac{x + y}{x - y}$

$\therefore \quad F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x - \lambda y} = \frac{\lambda(x + y)}{\lambda(x - y)} = \frac{x + y}{x - y} = \lambda^\circ F(x, y).$

Hence $F(x, y)$ is a homogeneous function of degree 0.

To solve the equation, we put $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

Now, putting the values of y and $\frac{dy}{dx}$ in (1), we get

$$v + x\frac{dv}{dx} = \frac{x + vx}{x - vx} = \frac{x(1 + v)}{x(1 - v)} = \frac{1 + v}{1 - v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1 + v}{1 - v} - v = \frac{1 + v - v + v^2}{1 - v} = \frac{1 + v^2}{1 - v}$$

$$\Rightarrow \quad \frac{dx}{x} = \frac{1 - v}{1 + v^2} dv \Rightarrow \int \frac{dx}{x} = \int \frac{1 - v}{1 + v^2} dv$$

$$\Rightarrow \quad \log|x| = \int \left(\frac{1}{1 + v^2} - \frac{v}{1 + v^2} \right) dv = \int \frac{1}{1 + v^2} dv - \frac{1}{2} \int \frac{2v}{1 + v^2} dv$$

$$= \tan^{-1} v - \frac{1}{2} \log(1 + v^2) + C_1$$

$$\Rightarrow \quad 2\log|x| = 2\tan^{-1} v - \log\left(1+\frac{y^2}{x^2}\right) + 2C_1$$

$$= 2\tan^{-1}\left(\frac{y}{x}\right) - \log\left(\frac{x^2+y^2}{x^2}\right) + 2C_1$$

$$= 2\tan^{-1}\left(\frac{y}{x}\right) - \{\log(x^2+y^2) - 2\log|x|\} + 2C_1$$

$$= 2\tan^{-1}\left(\frac{y}{x}\right) - \log(x^2+y^2) + 2\log|x| + 2C_1$$

$\Rightarrow \log(x^2+y^2) = 2\tan^{-1}\left(\frac{y}{x}\right) + C$ which is the required solution.

Or,

Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$.

Soln. Dividing throughout by y^2, the equation becomes $\frac{1}{y^2}\cdot\frac{dy}{dx} + \frac{1}{y}\cdot\frac{1}{x} = \frac{1}{x^2}$.

Put $z = \frac{1}{y}$ $\therefore \frac{dz}{dx} = -\frac{1}{y^2}\cdot\frac{dy}{dx}$.

Then the equation reduces to $-\frac{dz}{dx} + \frac{z}{x} = \frac{1}{x^2}$ i.e., $\frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}$

which is of the linear form in z.

Here $P = -\frac{1}{x}$ and $Q = -\frac{1}{x^2}$.

$\therefore$ I.F. $= e^{\int -\frac{1}{x}dx} = e^{-\log x} = e^{\log\frac{1}{x}} = \frac{1}{x}$.

Hence the general solution is $z \times \frac{1}{x} = \int -\frac{1}{x^2}\cdot\frac{1}{x}\,dx = -\int \frac{1}{x^3}\,dx$

$$= -\int x^{-3}\,dx = -\frac{x^{-3+1}}{-2} + C = \frac{1}{2x^2} + C$$

Now, putting the value of $z = \frac{1}{y}$, we get $\frac{1}{xy} = \frac{1}{2x^2} + C$.

11. Find the equations of the straight line perpendicular to the two lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$; $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ and passing through their point of intersection.

Soln. Here given equation of lines are

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1} \;\ldots\text{(i)} \quad \text{and} \quad \frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$$

$\therefore$ Points on equation (i) $P(-3r-1, 2r+3, r-2)$

and points on equation (ii) is $Q(\lambda, -3\lambda+7, 2\lambda-7)$

then for same point of intersection is $-3r-1 = \lambda \;\therefore\; \lambda + 3r = -1$

$2r+3 = -3\lambda + 7 \;\therefore\; -3\lambda - 2r = -4$

and $\quad r-2 = 2\lambda - 7 \;\therefore\; 2\lambda - r = 5$

Now, solving equation (iii) and (iv), then $r = -1, \lambda = 2$.

Now, putting these value in equation (v) then $2\times(-2)+1 = 5$. $5 = 5$

$\therefore$ P will be $(2, 1, -3)$.

Now, let d.c.'s of desired line in l, m, n and which is perpendicular to both the given equation.

then $-3l + 2m + n = 0$ ... (vi) and $l - 3m + 2n = 0$

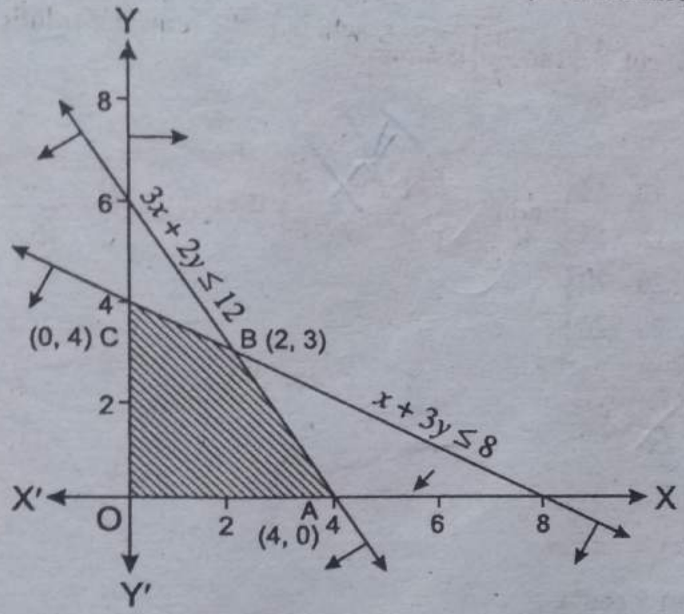
On solving them $\frac{l}{4+3} = \frac{-m}{-6-1} = \frac{n}{9-2} \Rightarrow \frac{l}{7} = \frac{m}{7} = \frac{n}{7}$. ... (vii)

Then d.c.'s of desired line will be $(7, 7, 7)$ and It's equation will be $\frac{x-2}{8} = \frac{y-1}{-1} = \frac{z+3}{5}$.

2. Minimize : $Z = -3x + 3y$

subject to $x + 2y \le 8;\ 3x + 2y \le 12;\ x \ge 0,\ y \ge 0.$

Soln. First of all, let us graph the feasible region on the system of inequations.



The shaded region in the figure above is the feasible region determined by the given system of constraints.

We observe that the feasible region $OABC$ is bounded. So we use corner point method to determine the minimum value of Z.

The co-ordinates of these corner points O, A, B, C are $(0, 0), (4, 0), (2, 3)$ and $(0, 4)$ respectively.

Now we evaluate $Z = -3x + 3y$ at each corner point.

Corner point	Corresponding value of $Z = -3x + 3y$	
(0, 0)	0	
(4, 0)	– 12	← Minimum
(2, 3)	3	
(0, 4)	12	

Hence, minimum value of Z is – 12 at the point (4, 0).

□